# VOLK Geometry

## Unit 10: Special Topics in Geometry

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**Student Name:______________________________**

**Block:_______**

# Logic and Conditional Statements

**Conditional Statement**

**If** **Hypothesis** **then** **Conclusion**

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**If** ***p*****,** **then** ***q***

$p \longrightarrow q$

**<u>or</u>** ***p*** **implies** ***q***

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**~*p*** **is read** **"not *p*"** **and means** **the opposite of *p***

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**Converse**

$q \to p$

**Inverse**

$\sim p \to \sim q$

**Contrapositive**

$\sim q \to \sim p$

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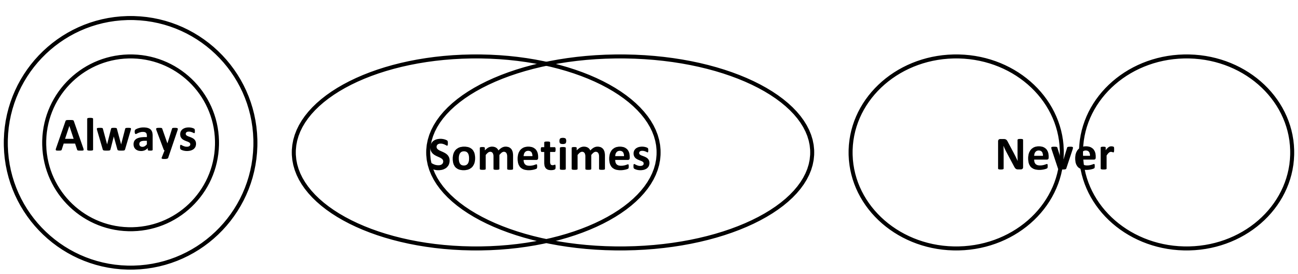


| Term | Definition |
| --- | --- |
| **Conditional Statement** | S_________________ that can be w___________________ in “if…, then” form |
| **Hypothesis** | Part of a c___________________ statement that f_________________ “if” |
| **Conclusion** | Part of a conditional s____________________ that follows “then” |
| **Negation** | D_________________ of a s______________________ formed by adding or removing the word ________________ from a statement |
| **Negate** | To add or remove the word ___________ from a statement to change its truth value from true to ____________________ or from false to ____________________ |
| **Converse** | S___________________ formed from a c__________________ statement by s_________________ the h_________________ and c__________________ |

| | |
|---|---|
| **Inverse** | S________________ formed from a c___________________ statement by<br><br>n____________________ the h____________________ and<br><br>c_____________________ |
| **Contrapositive** | S____________________ formed from a c___________________ statement by<br><br>s____________________ AND n____________________ the<br><br>h____________________ and c_____________________ |
| **Biconditional** | Statement combining a c______________________ statement and its<br><br>c_________________, using the phrase “if and o_____________ if” |

| | | | |
|---|---|---|---|
| ***p, q, r, s, t,* etc.** | Meaning: | | |
| $\rightarrow$ | Meaning: | $\vee$ | Meaning: |
| $\sim$ | Meaning: | $\therefore$ | Meaning: |
| $\wedge$ | Meaning: | $\leftrightarrow$ | Meaning: |

Use the following conditional statement to answer the problems: "If I win, then you don't lose."

1. Write the hypothesis. ________________________________
2. Write the conclusion. ________________________________
3. Negate the hypothesis. ________________________________
4. Negate the conclusion. ________________________________
5. Write the converse. ________________________________
6. Write the inverse. ________________________________
7. Write the contrapositive. ________________________________
8. Write the biconditional. ________________________________

Use the following conditional statement to answer the problems: "If elephants fly, then fish don't swim." Each answer should be a complete sentence, *not* symbols.

1. $p$ is the hypothesis. Write $p$. ________________________________
2. $q$ is the conclusion. Write $q$. ________________________________
3. $\sim p$ means "the negation of $p$." Write $\sim p$. ____________________
4. $\sim q$ means "the negation of $q$." Write $\sim q$. ____________________
5. (converse) $q \rightarrow p$ means "$q$ implies $p$" or "If $q$, then $p$." Write $q \rightarrow p$.

   ________________________________________________

6. (inverse) $\sim p \rightarrow \sim q$ means "Not $p$ implies not $q$" or "If not $p$, then not $q$." Write $\sim p \rightarrow \sim q$.

_______________________________________________

7. (contrapositive) $\sim q \rightarrow \sim p$ means "Not $q$ implies not $p$" or "If not $q$, then not $p$." Write $\sim q \rightarrow \sim p$. ________________________________

8. $p \wedge q$ means "$p$ and $q$." Write $p \wedge q$. ____________________

9. $p \vee q$ means "$p$ or $q$." Write $p \vee q$. ______________________

10. $\therefore p$ means "therefore $p$." Write $\therefore p$. ____________________

11. $p \leftrightarrow q$ means "$p$ if and only if $q$." Write $p \leftrightarrow q$. ___________

12. Write each of the following statements as a conditional statement. Then, circle the hypothesis, and underline the conclusion.

    a. Mark Twain wrote, "If you tell the truth, you don't have to remember anything."

    _______________________________________________

    b. Helen Keller wrote, "One can never consent to creep when one feels the impulse to soar."

    _______________________________________________

    c. Mahatma Ghandi wrote, "Freedom is not worth having if it does not include the freedom to make mistakes."

    _______________________________________________

d. Benjamin Franklin wrote, “Early to bed and early to rise makes a man healthy, wealthy, and wise.”

______________________________________________

13. Write the converse, inverse, and contrapositive for each of the following conditional statements. Determine whether each is true or false.

a. “If I win, then you don’t lose.”

Converse: ______________________________________

Inverse: ______________________________________

Contrapositive: ________________________________

True or false: _________________________________

b. “If two segments are congruent, then they have the same length.”

Converse: ______________________________________

Inverse: ______________________________________

Contrapositive: ________________________________

True or false: _________________________________

14. Use the Law of Detachment to reach a logical conclusion about the following statement: “If it is raining, then Sam and

Sarah will not go to the football game." This is a true conditional, and it is raining.

______________________________________________

15. Statement 1: "If two adjacent angles form a linear pair, then the sum of the measures of the angles is 180°."

Statement 2: "If the sum of the measures of two angles is 180°, then the angles are supplementary."

By the Law of Syllogism, which statement below follows from Statements 1 and 2?______________________________

a. If the sum of the measures of two angles is 180°, then the angles form a linear pair.

b. If two adjacent angles form a linear pair, then the sum of the measures of the angles is 180°.

c. If two adjacent angles form a linear pair, then the angles are supplementary.

d. If two angles are supplementary, then the sum of the measures of the angles is 180°.

16. Let *p*: you see lightning and *q*: you hear thunder. Write each of the following statements in symbolic notation:

a. If you see lightning, then you hear thunder.________________

b. If you hear thunder, then you see lightning.________________

c. If you don't see lightning, then you don't hear thunder.______

d. If you don't hear thunder, then you don't see lightning.______

17. Let *p*: two planes intersect and *q*: the intersection is a line. Write each of the following statements in symbolic notation:

a. If two planes don't intersect, then the intersection is a line.__

b. If the intersection is not a line, then two planes do not intersect.____________________________________________

18. Draw a Venn Diagram below for each of the following statements:

a. All squares are rhombi.

b. Some rectangles are squares.

c. No trapezoids are parallelograms.

19. John always listens to his favorite radio station, an oldies station, when he drives his car. Every morning he listens to his radio on the way to work. On Monday, when he turns on his car radio, it is playing country music. Make a list of valid conjectures to explain why his radio is playing different music.

20. $\angle M$ is obtuse. Make a list of conjectures based on that information.

21. Based on the table to the right, Marina concluded that when one of the two addends is negative, the sum is always negative. Write a counterexample for her conjecture.

| **Addends** | | **Sum** |
|---|---|---|
| −8 | −10 | −18 |
| −17 | −5 | −22 |
| 15 | −23 | −8 |
| −26 | 22 | −4 |

The Algebraic Properties of Equality can be used to solve

$$5x - 18 = 3x + 2$$

and to write a reason for each step, as shown in the table below.

| **Statement** | **Reason** |
|---|---|
| $5x - 18 = 3x + 2$ | Given |
| $2x - 18 = 2$ | Subtraction Property of Equality |
| $2x = 20$ | Addition Property of Equality |
| $x = 10$ | Division Property of Equality |

Using a table like this one, solve each of the following equations, and state a reason for each step.

22. $-2(-w + 3) = 15$

23. $p - 1 = 6$

24. $2r - 7 = 9$

25. $3(2t + 9) = 30$

26. Given $3(4v - 1) - 8v = 17$, prove $v = 5$.

Match each of the following conditional statements with a property.

- **A.** Multiplication Property
- **B.** Addition Property
- **C.** Distributive Property
- **D.** Substitution Property
- **E.** Symmetric Property
- **F.** Subtraction Property
- **G.** Transitive Property
- **H.** Reflexive Property
- **I.** Division Property

27. If $JK = PQ$ and $PQ = ST$, then $JK = ST$. _____

28. If $m\angle S = 30°$, then $5° + m\angle S = 35°$. _____

29. If $ST = 2$ and $SU = ST + 3$, then $SU = 5$. _____

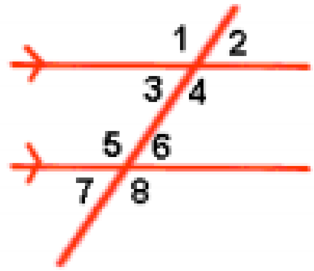
30. If $m\angle K = 45°$, then $3(m\angle K) = 135°$. _____

31. If $m\angle P = m\angle Q$, then $m\angle Q = m\angle P$. _____

# Algebraic Properties of Equality

| | |
|---|---|
| **Addition Property** | **If $a = b$, then $a + c = b + c$** |
| **Subtraction Property** | **If $a = b$, then $a - c = b - c$** |
| **Multiplication Property** | **If $a = b$, then $ac = bc$** |
| **Division Property** | **If $a = b$ and $c \neq 0$, then $a \div c = b \div c$** |
| **Reflexive Property** | **$a = a$** |
| **Symmetric Property** | **If $a = b$, then $b = a$** |
| **Transitive Property** | **If $a = b$ and $b = c$, then $a = c$** |
| **Substitution Property** | **If $a = b$, then $a$ can be substituted for $b$ in any equation or expression.** |
| **Distributive Property** | **$a(b + c) = ab + ac$** |

**Parallels: If lines are parallel …**




**Corresponding angles** are equal.
*m*<1=*m*<5, *m*<2=*m*<6, *m*<3=*m*<7, *m*<4=*m*<8
**Alternate Interior angles** are equal.
*m*<3=*m*<6, *m*<4=*m*<5
**Alternate Exterior angles** are equal.
*m*<1=*m*<8, *m*<2=*m*<7
**Same side interior angles** are supp.
*m*<3 + *m*<5=180, *m*<4 + *m*<6=180

**Coordinate Geometry Formulas:**
Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula:

$$(x, y) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Slopes and Equations:**

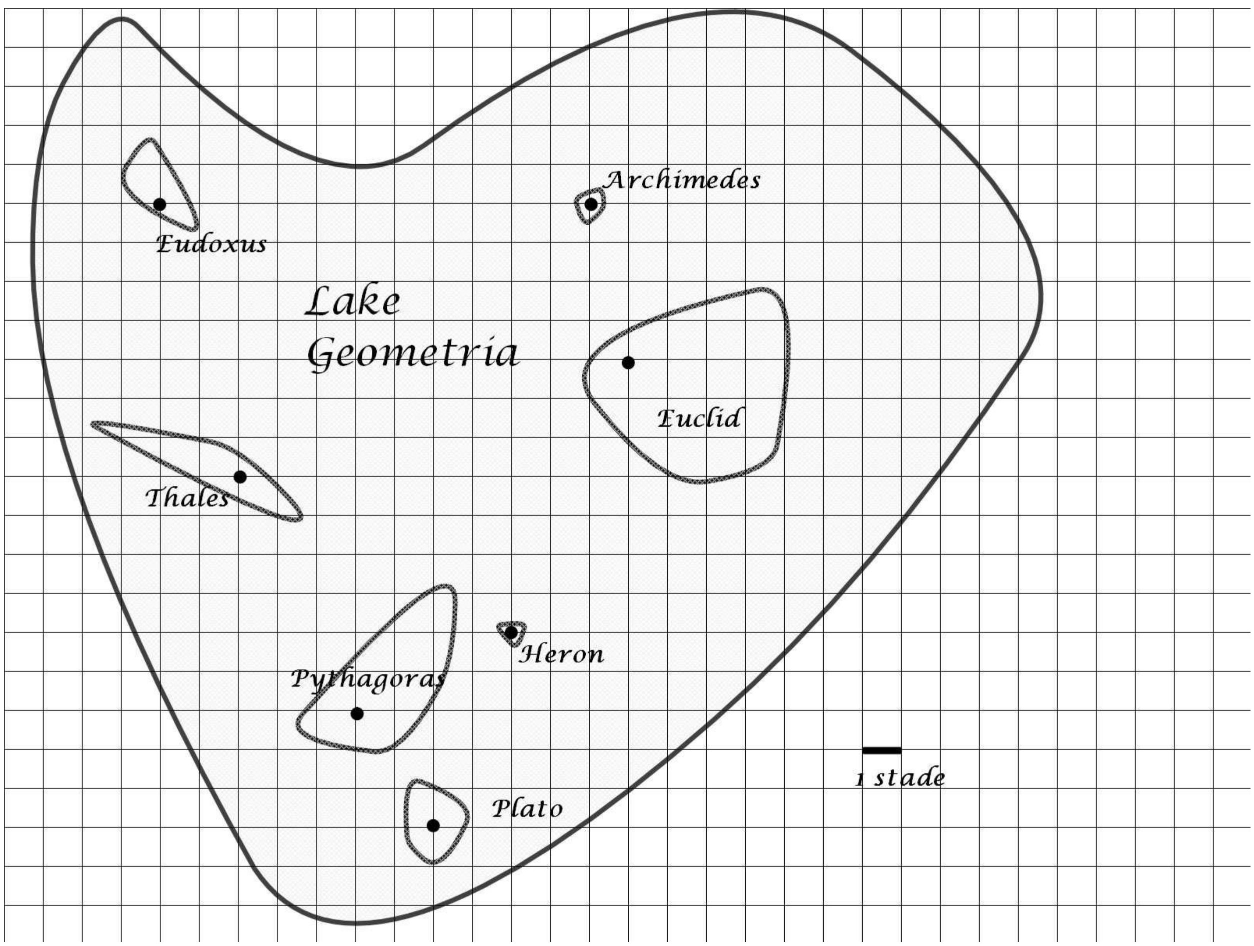
$$m = \frac{vertical\ change}{horizontal\ change} = \frac{y_2 - y_1}{x_2 - x_1}.$$

$y = mx + b$ slope-intercept

$y - y_1 = m(x - x_1)$ point-slope

## Lake Geometria

The islands of Lake Geometria are shown below. A cabin is marked on each island. The scale, using units called stades, is shown in the lower right. A stade measures about 600 feet, so Lake Geometria is not very big. Use the grid to help you answer the following questions. All island measurements should be made from cabin to cabin.




1. What is the distance in stades from Eudoxus to Archimedes? (Round to the nearest stade.) Describe how you found your answer.
2. What is the distance in stades from Thales to Euclid? (Round to the nearest stade.) Describe how you found your answer.

3. Which is closer to Thales—Pythagoras or Heron? Describe how you found your answer.

4. Find a point **in the water** that is the same distance from Archimedes and Eudoxus. Label the point *M*. Describe how you found this point.

5. Now find a point **on land** that is the same distance from Archimedes and Eudoxus. Label this point *N*.

6. How many points can you find that are the same distance from Archimedes and Eudoxus? Explain.

7. Find a point **on land** that is the same distance from Thales and Pythagoras. Is your point on the mainland or on an island? Does it have to be?

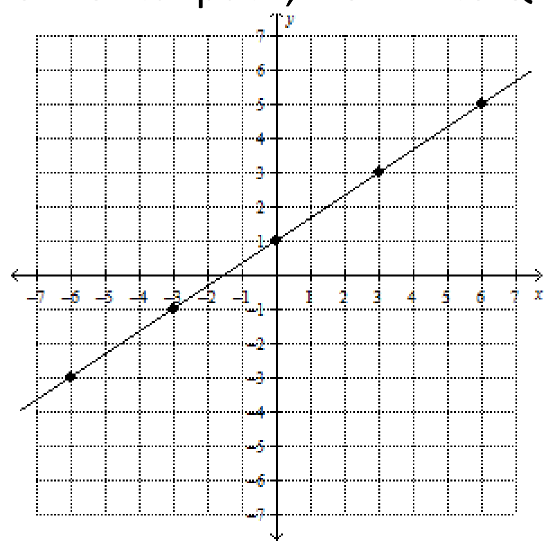
8. Groups staying on any of the islands are provided with solar-charged walkie-talkies, but their ranges are only about 1 mile. There are about 9 stades in a mile. With which islands could someone staying at the cabin on Thales expect to be able to communicate? Show your work, or explain how you found your answer.

9. Estimate the shortest distance from Thales to the mainland. (You may use stades, feet, or miles.) Show your work or explain how you found your answer.

10. Estimate how many miles wide Lake Geometria is at its widest point. Explain how you found your answer.

# Slope of a Line

1. Select two points on the graph of the line below, and label one *P* and the other *Q*.
2. Draw a vertical path followed by a horizontal path, from *P* to *Q*.




3. What is the vertical distance?

4. What is the horizontal distance?

5. What is the ratio of the vertical distance divided by the horizontal distance?

6. What term do we use for this ratio?

7. The formula for finding the slope of the line through two points is $\frac{y_2 \quad y_1}{x_2 \quad x_1}$. Use this formula and the points you used above to compute the slope of the line. Do you get the same answer?

8. Find three other points on the line through *R* (−2, 4) with the same slope as the line above. Graph the line. Does it appear that the two lines are parallel, perpendicular, or neither?

9. Find three other points on the line through *R* (−2, 4) with slope $-\frac{2}{3}$. Graph the line.

10. Use a corner of a piece of paper to check the angle formed by the two lines. What does that angle appear to be? Does it appear that the two lines are parallel, perpendicular, or neither?

11. Multiply the slopes of the two lines together. What is the product?

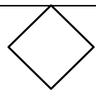
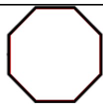
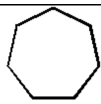
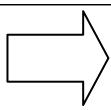
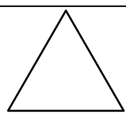
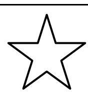
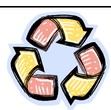
# Symmetry

- A figure has **line symmetry** if there is a line that divides the figure into mirror images.

- A figure has **rotational symmetry** if it looks the same when rotated some angle measure less than 360 degrees. Its **order of rotational symmetry** is the number of positions a figure can be rotated, without changing the way it looks. It has n° rotational symmetry (for example 90° rotational symmetry) if it looks the same when rotated n°.

- A figure has **point symmetry** if it looks the same upside-down, or rotated 180 degrees. Such a figure also has rotational symmetry of order 2.

Identify the apparent number of lines of symmetry and order of rotational symmetry for each figure. Assume polygons that appear to be regular are. (on the next page)

| Shape | Number of Lines of Symmetry | Order of Rotational Symmetry | Degrees of Rotational Symmetry | Does this figure have point symmetry? |
|---|---|---|---|---|
|  | 4 | 4 | 90, 180, 270, 360 | yes |
|  | | | | |
|  | | | | |
|  | | | | |
|  | | | 360 | |
|  | | | | |
|  | | | | |
|  | | | | |
| **M** | | | | |
| **A** | | | | |
| **T** | | | | |
| **H** | | | | |
| **S** | | | | |

1 **Which statement is the converse of the following statement?**

**If two angles are complementary, then the sum of their angle measures is 90°.**

**A** If two angles are not complementary, then the sum of their angle measures is not 90°.

**B** Two angles are complementary if and only if the sum of their angle measures is 90°.

**C** If the sum of the measures of two angles is not 90°, then the angles are not complementary.

**D** If the sum of the measures of two angles is 90°, then the angles are complementary.

2 ***If two triangles are both equilateral, then they are similar.***

**Which of the following best describes the *contrapositive* of the assertion above?**

**A** If two triangles are not both equilateral, then they are not similar.

**B** Two triangles are similar if and only if they are both equilateral.

**C** If two triangles are not similar, then they are not both equilateral.

**D** If two triangles are similar, then they are both equilateral.

3 **You are told that a conditional statement is false. Which statement is also false?**

**F** inverse

**G** contrapositive

**H** converse

**J** conclusion

4 **"Switch the hypothesis and the conclusion." This is a procedure for constructing which of the following?**

**A** The inverse

**B** The converse

**C** The contrapositive

**D** None of the above

5 **What is the inverse of the given statement?**

**GIVEN: If you do not enter the contest, you cannot win the contest.**

**F** If you enter the contest, you can win the contest.

**G** If you cannot win the contest, do not enter the contest.

**H** If you enter the contest, you cannot win the contest.

**J** If you can win the contest, then enter the contest.

6 **Given that the inverse of a statement is true, what other statement is true?**

**F** The original statement

**G** The hypothesis

**H** The converse

**J** The contrapositive

7 **If an argument containing a conditional statement among the premises is valid, then the argument remains valid when the conditional statement is replaced with which of the following?**

**A** Its negation

**B** Its inverse

**C** Its converse

**D** Its contrapositive

8 **If a conditional statement seems difficult to prove, one can instead try to prove which equivalent statement?**

**F** The negation

**G** The inverse

**H** The converse

**J** The contrapositive

9 **Let $p$ be "it is raining," let $q$ be "it is thundering," and let $r$ be "we cannot swim." What is $\sim r \rightarrow \sim q$?**

**A** If it is thundering, then we cannot swim.

**B** If we can swim, then it is not thundering.

**C** If we cannot swim, then it is not thundering.

**D** If we can swim, then it is thundering.

10 **Which of the following argument forms is *invalid*?**

**A** $q$
$p \rightarrow q$
Therefore, $p$

**B** $p$
$p \rightarrow q$
Therefore, $q$

**C** $\sim q$
$p \rightarrow q$
Therefore, $\sim p$

**D** $p \rightarrow q$
$q \rightarrow r$
Therefore, $p \rightarrow r$

11 **Which of the following argument forms is *valid*?**

**F** $p \rightarrow q$
$\sim p$
Therefore, $\sim q$

**G** $p \rightarrow q$
$\sim q$
Therefore, $\sim p$

**H** $p \rightarrow q$
$q$
Therefore, $p$

**J** $p \rightarrow q$
$p \rightarrow r$
Therefore, $q \rightarrow r$

12 **Let $p$ be "it is raining," let $q$ be "it is thundering," and let $r$ be "we cannot swim." The statement $\sim p \rightarrow \sim r$ could be**

G

**F** The inverse of $r \rightarrow p$

**G** The inverse of $\sim r \rightarrow \sim p$

**H** The contrapositive of $r \rightarrow p$

**J** The converse of $r \rightarrow p$

13 **Which of the following is guaranteed to make a conditional statement true?**

**A** A true hypothesis

**B** A false hypothesis

**C** A false conclusion

**D** None of the above

14 **Let $p$ be "the election was stolen," let $q$ be "the ballots were tampered with," and let $r$ be "officials lost control of the ballots." Translate the following argument into symbolic form.**

***If officials did not lose control of the ballots, then ballots were not tampered with.***

***And if the ballots were not tampered with, then the election was not stolen.***

***Therefore, if the election was stolen, then officials lost control of the ballots.***

**A** $\sim r \rightarrow \sim q$
$\sim q \rightarrow \sim p$
Therefore, $p \rightarrow r$

**B** $\sim p \rightarrow \sim q$
$\sim q \rightarrow \sim r$
Therefore, $r \rightarrow p$

**C** $\sim q \rightarrow r$
$r \rightarrow \sim p$
Therefore, $p \rightarrow q$

**D** $r \rightarrow \sim p$
$p \rightarrow \sim q$
Therefore, $q \rightarrow \sim r$

15 **Which of the following is guaranteed to make a conditional statement false?**

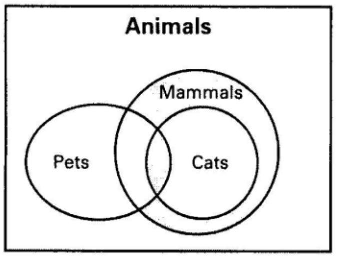
**F** A true hypothesis

**G** A false hypothesis

**H** A false conclusion

**J** None of the above

16 **Which of the following statements is represented by the Venn diagram?**




**A** No pets are mammals.

**B** All cats are mammals.

**C** All cats are pets.

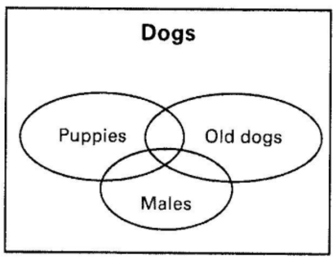
**D** All mammals are cats.

17 **Some puppies are males.**
**Some males are old dogs.**

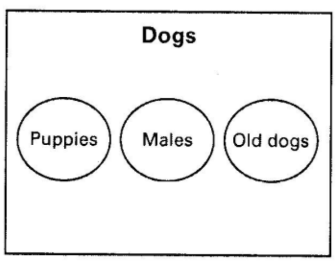
**Therefore, some puppies are old dogs.**

**Which of the following Venn diagrams shows the above argument to be invalid?**

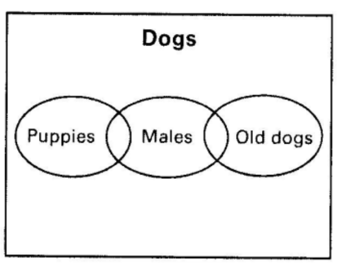
**F**



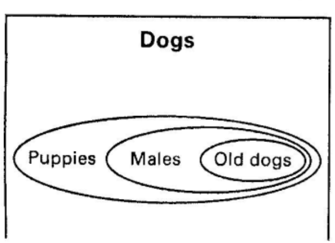

**G**



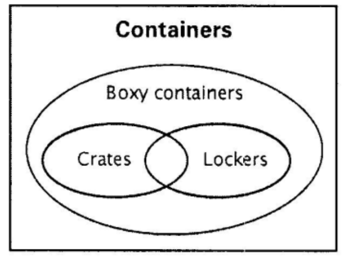

**H**




**J**




18 **Which of the following statements is *not* represented by the Venn diagram?**




**F** No boxy containers are crates.

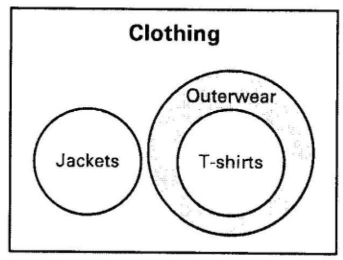
**G** All lockers are boxy containers.

**H** Some crates are lockers.

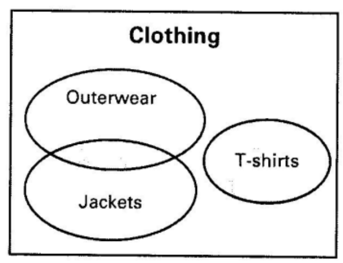
**J** Some boxy containers are lockers.

19 **Which of the following Venn diagrams represents the statement "All jackets are outerwear"?**

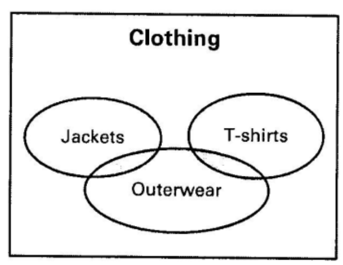
**A**



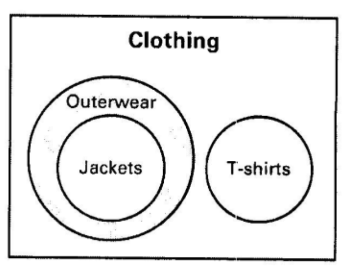

**B**




**C**




**D**



**20** **Line *c* intersects both line *a* and line *b*. What else must be true for line *c* to be considered the transversal of lines *a* and *b*?**

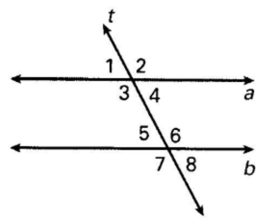
**A** Lines *a* and *b* must be parallel.

**B** Lines *a* and *b* must intersect.

**C** Line *c* must not intersect any other lines.

**D** Nothing else is needed.

**Use the figure below for questions 21 and 22 Line *t* intersects lines *a* and *b*.**




**21** **Which angle has to have the same measure as $\angle 2$ for lines *a* and *b* to be parallel?**

**A** $\angle 1$ **C** $\angle 7$

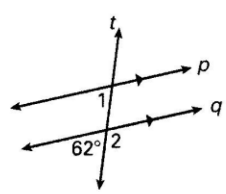
**B** $\angle 3$ **D** $\angle 8$

**22** **Which angle has to be supplementary to $\angle 4$ for lines *a* and *b* to be parallel?**

**F** $\angle 6$ **H** $\angle 5$

**G** $\angle 3$ **J** $\angle 8$

**23** **Line *t* intersects lines *p* and *q*.**




**Which statement must be true about $\angle 1$ and $\angle 2$ in order for line *p* and line *q* to be parallel?**

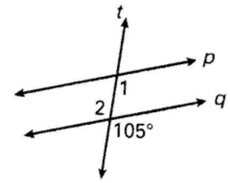
**A** Their measures must sum to 62°.

**B** Their measures must sum to 118°.

**C** Their measures must be supplementary.

**D** Their measures must be equal.

**24** **Line *t* intersects lines *p* and *q*.**




**Which statement must be true about $\angle 1$ and $\angle 2$ in order for line *p* and line *q* to be parallel?**

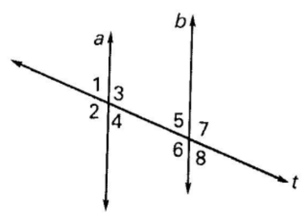
**F** Their measures must be equal.

**G** Their measures must be supplementary.

**H** Their measures must sum to 105°.

**J** Their measures must sum to 75°.

**Use the figure below for questions 25 and 26 Line *t* intersects lines *a* and *b*.**




**25** **Which angle has to have the same measure as $\angle 5$ for lines *a* and *b* to be parallel?**

**A** $\angle 3$ **C** $\angle 8$

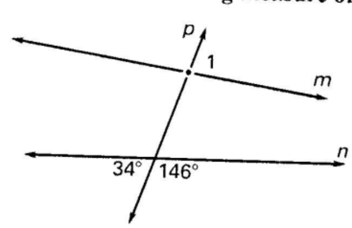
**B** $\angle 1$ **D** $\angle 2$

**26** **Which angle has to be supplementary to $\angle 3$ for lines *a* and *b* to be parallel?**

**F** $\angle 6$ **H** $\angle 7$

**G** $\angle 8$ **J** $\angle 4$

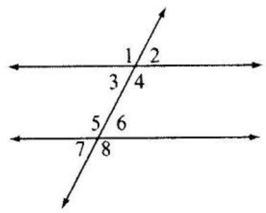
**27** **If line *m* is rotated about its intersection with line *p*, until line *m* is parallel to line *n*, what is the resulting measure of $\angle 1$?**




**F** 34° **H** 90°

**G** 146° **J** 112°

28 **Classify the pair $\angle 1$ and $\angle 8$.**



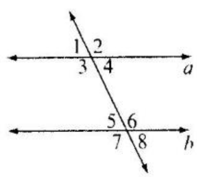

**A** Corresponding angles

**B** Alternate interior angles

**C** Alternate exterior angles

**D** Consecutive interior angles

**Use the figure below for questions** 29, 30, 31




29 **Choose the reason the statement "If $m\angle 3 = 115°$, then $m\angle 5 = 65°$" is true.**

**A** Alternate Interior Angles Theorem

**B** Alternate Exterior Angles Theorem

**C** Consecutive Interior Angles Theorem

**D** Vertical Angles Theorem

30 **Choose the reason the statement "If $m\angle 1 = 65°$, then $m\angle 5 = 65°$" is true.**

**F** Alternate Interior Angles Theorem

**G** Alternate Exterior Angles Theorem

**H** Consecutive Interior Angles Theorem

**J** Corresponding Angles Postulate

31 **If $m\angle 6 = 115°$, then $m\angle 3 = ?$**

**A** 65°

**B** 115°

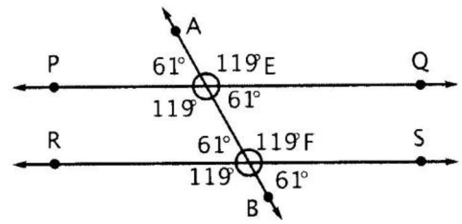
**C** 180°

**D** 90°

32 **Use the diagram to answer the question below.**

$\angle PEA \cong \angle RFE$

**Prove that line $PQ$ is parallel to line $RS$.**




**What reason can be used to prove that lines $PQ$ and $RS$ are parallel?**

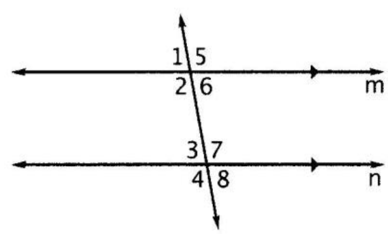
**F** The distance between $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ is the same.

**G** Corresponding angles are congruent.

**H** Supplementary angles are congruent.

**J** $\overleftrightarrow{AB}$ is a perpendicular transversal.

33 **Examine the diagram below, where lines $m$ and $n$ are parallel. Which is a valid conclusion and valid reasoning based on the diagram?**



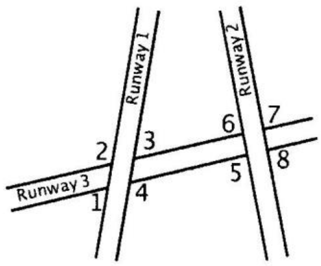

**F** $\angle 2$ and $\angle 8$ are congruent because corresponding and vertical angles are congruent.

**G** The measures of $\angle 1$ and $\angle 4$ have a sum of 180° because same-side exterior angles are supplementary.

**H** $\angle 5$ and $\angle 3$ are congruent because alternate interior angles are congruent.

**J** $\angle 6$ and $\angle 7$ are congruent because same-side interior angles are congruent.

**Use the figure below for questions 34 & 35 In the figure, runway 3 crosses runways 1 and 2 and acts as a transversal.**




34 **Which pair of angles formed by the runways must be congruent?**

**A** $\angle 2$ and $\angle 4$

**B** $\angle 1$ and $\angle 2$

**C** $\angle 2$ and $\angle 8$

**D** $\angle 5$ and $\angle 6$

35 **If runways 1 and 2 are to be parallel, what must be true by the Corresponding Angles Postulate?**

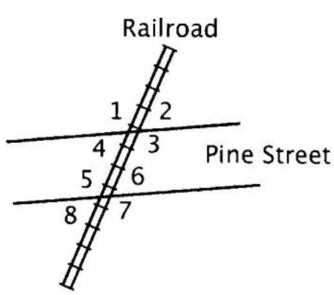
**F** $\angle 4 \cong \angle 8$

**G** $\angle 1 \cong \angle 7$

**H** $\angle 2$ and $\angle 7$ are supplementary.

**J** $\angle 1$ and $\angle 8$ are supplementary.

**Use the figure below for questions 36-38 The railroad track represents a transversal to the lines that represent the sides of Pine Street. The sides of the street are parallel.**




36 **Which statement is justified by the Alternate Exterior Angles Theorem?**

**A** $\angle 3 \cong \angle 5$

**B** $\angle 2 \cong \angle 8$

**C** $\angle 1$ and $\angle 8$ are supplementary.

**D** $\angle 4$ and $\angle 5$ are supplementary.

37 **Which statement is justified by the Consecutive Interior Angles Theorem?**

**F** $\angle 4 \cong \angle 6$

**G** $\angle 1 \cong \angle 7$

**H** $\angle 3$ and $\angle 6$ are supplementary.

**J** $\angle 2$ and $\angle 7$ are supplementary.

38 **Which of the following statements is correct?**

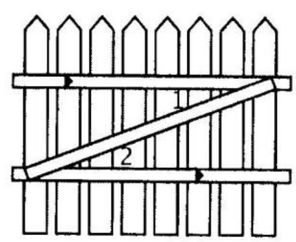
**A** $\angle 1$ and $\angle 6$ are complementary.

**B** $\angle 1$ and $\angle 6$ are supplementary.

**C** $\angle 1$ and $\angle 6$ are congruent.

**D** $\angle 1$ and $\angle 6$ are similar.

39 **A wooden gate has *z*-shaped boards for support, as shown.**




**Which of the following statements is true?**

**F** $m\angle 1 + m\angle 2 = 180$

**G** $m\angle 1 + m\angle 2 = 90$

**H** $m\angle 2 - m\angle 1 = 2(m\angle 1)$

**J** $m\angle 1 + m\angle 2 = 2(m\angle 2)$

40 On a map, two cities are located at (2, 4) and (−2, 2). What is the distance between the cities on the map?

**A** $2\sqrt{3}$ units

**B** $2\sqrt{5}$ units

**C** 12 units

**D** 20 units

41 The endpoints of $\overline{MN}$ are $M(-3, -9)$ and $N(4, 8)$. What is the approximate length of $\overline{MN}$?

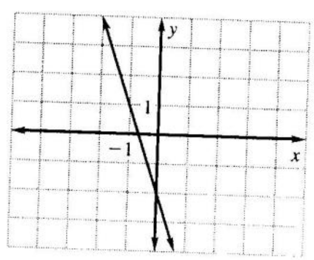
**F** 1.4 units

**G** 7.2 units

**H** 13 units

**J** 18.4 units

42 What rate of change is illustrated in the graph?




**A** $-3$

**B** $-\frac{1}{3}$

**C** $\frac{1}{3}$

**D** 3

43 A passenger train is climbing a mountain that has a slope of 0.4 as it takes riders on a tour of a wildlife refuge. After traveling a vertical distance of 40 feet, how many feet has the train traveled horizontally?

**F** 100 ft

**G** 60 ft

**H** 36 ft

**J** 16 ft

44 Find the coordinates of the other endpoint of a segment with endpoint $X(-2, 3)$ and midpoint $M(1, -2)$.

**A** (4, −7)

**B** (−4, 7)

**C** (0, −1)

**D** (−5, 8)

45 Carlos is walking in a straight line from the soccer field at point $S(-2.23, 4.71)$ to his house at point $H(4.2, 9.92)$. At what point will he be halfway home?

**F** (0.985, 2.605)

**G** (0.985, 7.315)

**H** (3.215, 2.605)

**J** (3.215, 7.315)

46 Line segments $\overline{RS}$ and $\overline{TU}$ are congruent. Their coordinates are $R(0, 2)$, $S(3, 6)$, $T(6, 1)$, $U(10, y)$. Find the value of $y$.

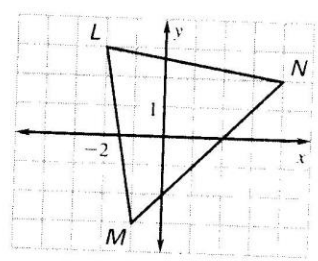
**A** 2

**B** 3

**C** 4

**D** 5

47 To the nearest hundredth of a unit, what is the length of the segment joining the midpoints of $\overline{LM}$ and $\overline{LN}$?




**F** 2.24

**G** 3.54

**H** 4.16

**J** 4.48

48. **Which lines are perpendicular?**

**Line 1:** $2x + y = 4$

**Line 2:** $y = x - 7$

**Line 3:** $\frac{1}{2}x - y = -3$

**A** Lines 1 and 2

**B** Lines 1 and 3

**C** Lines 2 and 3

**D** None of the lines are perpendicular.

49. **Which of the following statements are true about lines $w$, $n$, $p$, and $z$?**

$w: y = \frac{3}{2}x + 2$

$n: y = \frac{2}{3}x + 6$

$p: y = -\frac{3}{2}x - 3$

$z: y = \frac{2}{3}x + 1$

**I.** $w \perp p$ **II.** $n \parallel z$ **III.** $z \perp p$

**F** I only

**G** II only

**H** III only

**J** II and III

50. **Which pair of lines are perpendicular?**

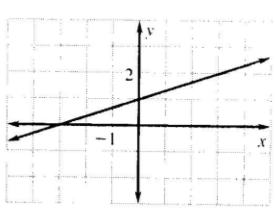
**A** Line 1: $(8, 12), (7, -5)$
Line 2: $(-9, 3), (8, 2)$

**B** Line 1: $(3, -4), (-1, 4)$
Line 2: $(2, 7), (5, 1)$

**C** Line 1: $(-3, 1), (-7, -2)$
Line 2: $(2, -1), (8, 4)$

**D** Line 1: $(-1, 3), (4, 1)$
Line 2: $(-2, -1), (3, -3)$

51. **Grant wrote an equation of a line through the point (4, 1) that is perpendicular to the one shown. What other point lies on his line?**




**F** $(2, 7)$ **H** $(-1, 12)$

**G** $(3, 3)$ **J** $(4, 5)$

52. **If two different lines with equations $y = m_1x + b_1$ and $y = m_2x + b_2$ are parallel, which of the following must be true?**

**F** $b_1 = b_2$ and $m_1 \neq m_2$

**G** $b_1 \neq b_2$ and $m_1 = m_2$

**H** $b_1 \neq b_2$ and $m_1 \neq m_2$

**J** $b_1 = b_2$ and $m_1 = m_2$

53. **Which equation is an equation of the line parallel to $3x + 4y = 7$ that passes through the point $(2, -1)$?**

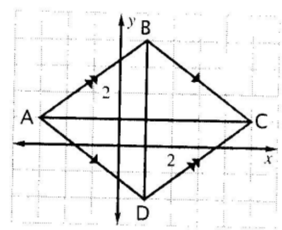
**A** $y = \frac{4}{3}x - \frac{11}{3}$

**B** $y = -\frac{4}{3}x + \frac{5}{3}$

**C** $y = \frac{3}{4}x - \frac{5}{2}$

**D** $y = -\frac{3}{4}x + \frac{1}{2}$

54. **Figure *ABCD* is a parallelogram. Which statement would prove that parallelogram *ABCD* is a rhombus?**



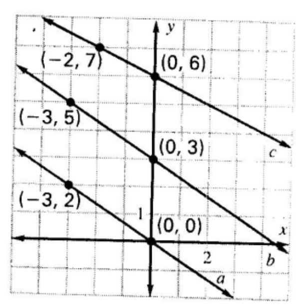

**F** $AB = CD$

**G** $AC = BD$

**H** slope $\overline{AB}$ = slope $\overline{CD}$

**J** $(\text{slope } \overline{AC})(\text{slope } \overline{BD}) = -1$

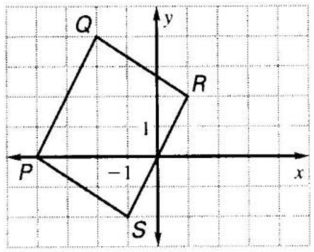
55. **Which lines are parallel?**




**A** $a \parallel c$ **C** $a \parallel b$

**B** $b \parallel c$ **D** $a \parallel b \parallel c$

56 **Which type of symmetry does the figure possess?**




**A** 180° clockwise rotation about the point $\left(\frac{-3}{2}, 1\right)$.

**B** 90° clockwise rotation about the point $\left(\frac{-3}{2}, 1\right)$.

**C** Reflection across a line through points $Q$ and $S$.

**D** Reflection across a line through points $P$ and $R$.

57 **A regular pentagon has many symmetries. Two of them are __________ and __________ rotation symmetry.**

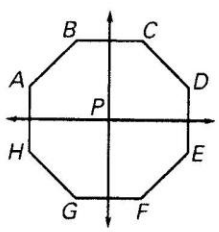
**F** 72° clockwise; 144° counterclockwise

**G** 36° clockwise; 108° counterclockwise

**H** 90° clockwise; 270° counterclockwise

**J** 60° clockwise; 240° counterclockwise

58 **In the figure below, which segment represents a 90° clockwise rotation of segment $AB$ about $P$?**



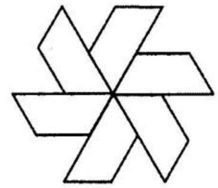

**F** $\overline{BC}$

**G** $\overline{EF}$

**H** $\overline{HG}$

**J** $\overline{CD}$

59 **The design below is made of congruent isosceles trapezoids**



**Each of the trapezoids has _____________ symmetry, and the figure as a whole has _____________ symmetry. Choose the *best* pair of answers to fill in the blanks.**

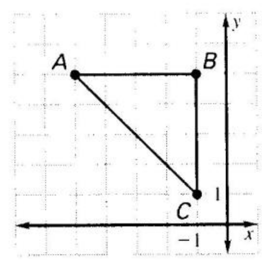
**A** Reflection; reflection

**B** Reflection; rotation

**C** Rotation; reflection

**D** Rotation; rotation

60 **Which of the following *best* describes the line of reflection symmetry in the figure below?**




**F** Vertical line through the point $(-2, 3)$

**G** Horizontal line through the point $(-2, 3)$

**H** Diagonal line through point $B$

**J** Diagonal line through points $A$ and $C$

61 **The shortest distance between point $W$ and a line of reflection is 64 inches. What will be the distance between $W$ and its image $W'$?**

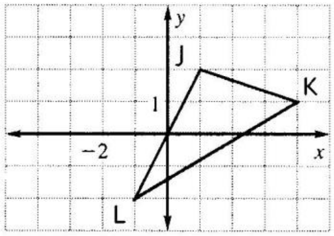
**F** 8 in.

**G** 32 in.

**H** 64 in.

**J** 128 in.

62 **Emily wants to transform $\triangle JKL$ so that $\triangle J'K'L'$ has the coordinates $J'(-3, 5)$, $K'(0, 4)$, and $L'(-5, 1)$.**




**Which transformation should she perform?**

**A** Translate $\triangle JKL$ 4 units left and 3 units up.

**B** Rotate $\triangle JKL$ counterclockwise 90°.

**C** Reflect $\triangle JKL$ across the $x$-axis.

**D** Reflect $\triangle JKL$ across the $y$-axis.

63 **Lauren is working on a problem where she must rotate point $J$ 90° clockwise about the origin. If the coordinates of $J$ are $(2, -7)$, where should she plot the final image?**

**F** $(2, 7)$

**G** $(-2, 7)$

**H** $(7, 2)$

**J** $(-7, -2)$

64 **In quadrilateral $RSTU$, the coordinates of $R$ are $(3, -5)$. What are the coordinates of the image of $R$ after a rotation of 180° counterclockwise?**

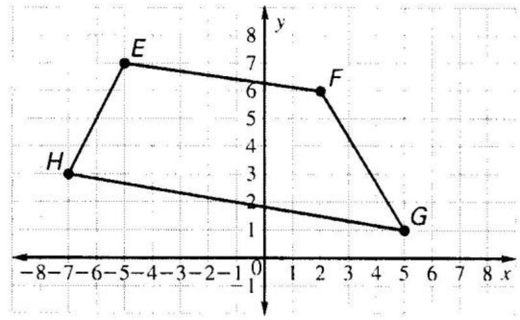
**A** $R'(-3, -5)$

**B** $R'(-5, -3)$

**C** $R'(5, 3)$

**D** $R'(-3, 5)$

65 **If quadrilateral $EFGH$ was reflected over the $x$-axis to form $E'F'G'H'$, what would be the coordinates of $E'$?**




**F** $(5, -7)$ **H** $(-5, 7)$

**G** $(5, 7)$ **J** $(-5, -7)$

66 **One vertex of a triangle is located at the point $P(-2, 7)$. If a scale factor of 2.2 is used for a dilation of the triangle, where will the image point $P'$ be located?**

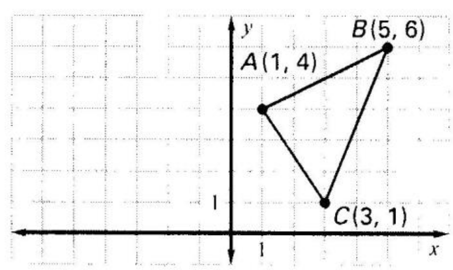
**A** $(-4.4, 15.4)$

**B** $(-4.2, 4.8)$

**C** $(0.2, 9.2)$

**D** $(-0.9, 3.2)$

67 **$\triangle ABC$ is to be reflected across the $y$-axis. What will be the coordinates of $B'$?**




**F** $(5, -6)$

**G** $(-1, 4)$

**H** $(-5, 6)$

**J** $(3, -1)$

68 **In constructing a line segment congruent to a given line segment, what is the purpose of drawing an arc that intersects the second segment?**

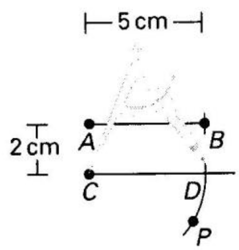
**A** To set the compass at the length of the original segment

**B** To mark the end of the second segment

**C** To establish the angle between the two segments

**D** To ensure that the two segments are parallel

69 **The figure shows the construction of a line segment congruent to a given line segment. What is the distance from point *C* to point *P*?**




**F** 2 cm

**G** 3 cm

**H** 5 cm

**J** 7 cm

70 **In constructing a line segment $\overline{CD}$ congruent to a given line segment $\overline{AB}$. what is the purpose of placing the compass at *C*?**

**A** Preparing to measure the distance from point *C* to point *A*

**B** Preparing to set the compass to the length of $\overline{AB}$

**C** Preparing to connect points *C* and *D*

**D** Preparing to mark point *D*

71 **Some of the steps for constructing a line segment $\overline{CD}$ congruent to a given line segment $\overline{AB}$ are listed below. Which of these is the *first* step?**

**F** Place the compass at point *C*.

**G** Mark point *D* on the new segment.

**H** Label one end of the new segment as point *C*.

**J** Draw a segment longer than $\overline{AB}$.

72 **For which of the following would be it be useful to know how to construct a line segment congruent to a given line segment? Choose the *best* answer.**

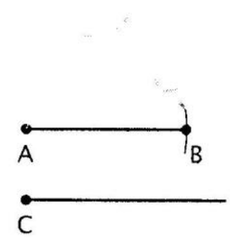
**A** To draw a perfect circle

**B** To draw a quadrilateral with two congruent sides

**C** To draw a trapezoid with two perfectly parallel sides

**D** To draw a perfect rectangle

73 **What is being done in the figure, as part of constructing a line segment congruent to a given line segment?**



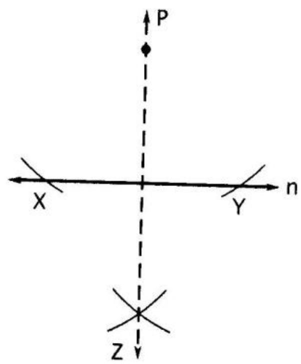

**F** The location of point *A* is being determined.

**G** The location of point *B* is being determined.

**H** The compass is being set to the length of segment $\overline{AB}$.

**J** Segment $\overline{AB}$ is being drawn.

For questions 74 & 75 use the figure below. It shows the construction of a perpendicular to given line $n$ through given point $P$. All arcs were drawn with the same compass setting.




74 **Suppose that distance $PY$ is 6.2 cm. What is distance $XZ$?**

**A** 12.4 cm

**B** 8.8 cm

**C** 6.2 cm

**D** Cannot be determined

75 **Suppose that distance $XZ$ is 4.9 cm. What is distance $PZ$?**

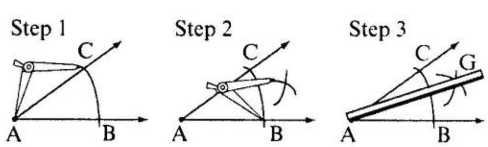
**F** 9.8 cm

**G** 6.9 cm

**H** 4.9 cm

**J** Cannot be determined

**figure shows the construction of the bisector of $\angle CAB$.**




76 **Which distances can you assume are equal?**

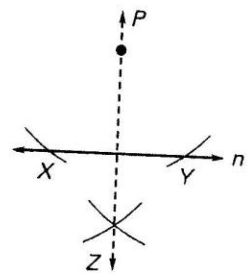
**A** $AC$ and $AG$

**B** $AC$ and $AB$

**C** $AG$ and $AB$

**D** $AC$ and $BC$

**The figure shows an attempt to construct a perpendicular to line $n$ from point $P$. Distances $PX$ and $PY$ are equal and distances $XZ$ and $YZ$ are equal, but $PX$ does not equal $XZ$.**




77 **What is wrong with this construction?**

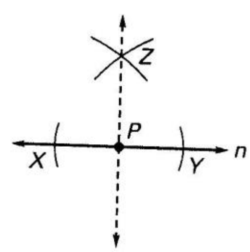
**F** Line $\overleftrightarrow{PZ}$ will not be perpendicular to line $n$.

**G** Line $\overleftrightarrow{PZ}$ will not bisect segment $\overline{XY}$.

**H** The perpendicular will not pass exactly through point $P$.

**J** Nothing is wrong. The construction will still work.

**The figure shows the construction of a perpendicular to given line $n$ through given point $P$.**




78 **Suppose that distance $XP$ is 7.3 cm. What is distance $YZ$?**

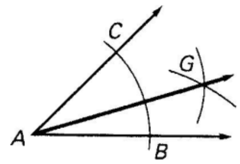
**A** 16.3 cm

**B** 14.6 cm

**C** 7.3 cm

**D** Cannot be determined

79 **What went wrong in the construction below?**




**F** Arc $\widehat{BC}$ is not centered at $A$.

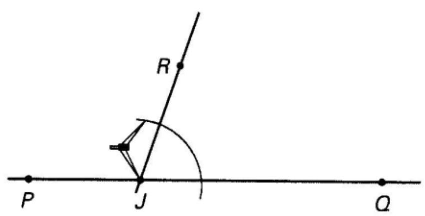
**G** Distance $AG$ should not be longer than distances $AC$ and $AB$.

**H** The two arcs through $G$ have different radii.

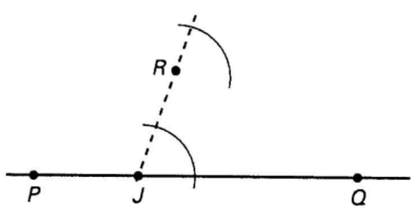
**J** Nothing is wrong; the construction is done correctly.

80 **George is constructing a line parallel to line $PQ$ that passes through point $R$. Which of the following should be his first step?**

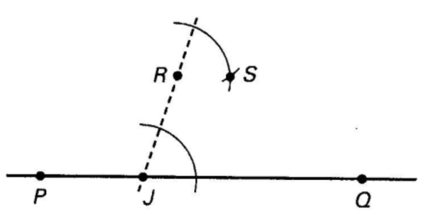
**F**



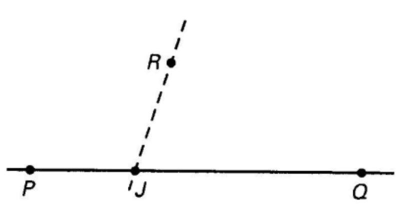

**G**



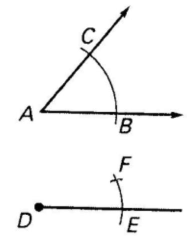

**H**




**J**




81 **What went wrong with the following attempt to copy $\angle CAB$?**



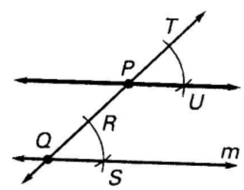

**A** The reference point for the compass should be $D$, not $E$.

**B** Point $F$ should be drawn on the original angle, not the copy.

**C** The compass being used to draw point $F$ is not set at distance $BC$.

**D** Angle $CAB$ was drawn wider than it should be.

82 **The figure shows the construction of a line through point $P$ parallel to line $m$.**



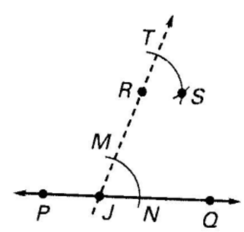

**If $m\angle RQS = 50°$, what is $m\angle TPU$?**

**F** 50°

**G** 65°

**H** 80°

**J** 130°




83 **The figure shows point $S$ being drawn to meet certain conditions. What are the conditions?**

**A** $RS = JN$ and $RN = SJ$

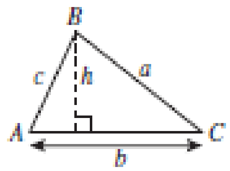
**B** $TS = MN$ and $SP = MQ$

**C** $RS = JN$ and $TS = MN$

**D** $RJ = SN$ and $SP = MQ$

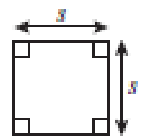
# Geometry Formula Sheet
# 2009 *Mathematics Standards of Learning*

## Geometric Formulas

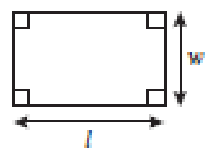



$$A = \frac{1}{2}bh$$

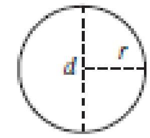
$$A = \frac{1}{2}ab \sin C$$



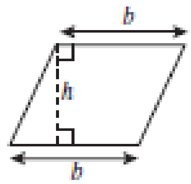

$$p = 4s$$
$$A = s^2$$



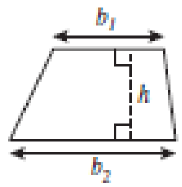

$$p = 2l + 2w$$
$$A = lw$$



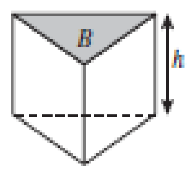

$$C = 2\pi r$$
$$C = \pi d$$
$$A = \pi r^2$$



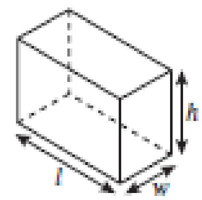

$$A = bh$$



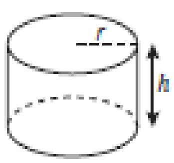

$$A = \frac{1}{2}h(b_1 + b_2)$$



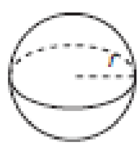

$$V = Bh$$
$$L.A. = hp$$
$$S.A. = hp + 2B$$



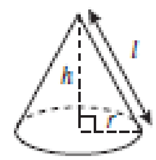

$$V = lwh$$
$$S.A. = 2lw + 2lh + 2wh$$



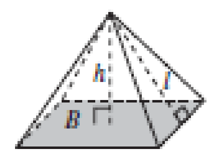

$$V = \pi r^2 h$$
$$L.A. = 2\pi rh$$
$$S.A. = 2\pi r^2 + 2\pi rh$$




$$V = \frac{4}{3}\pi r^3$$
$$S.A. = 4\pi r^2$$




$$V = \frac{1}{3}\pi r^2 h$$
$$L.A. = \pi rl$$
$$S.A. = \pi r^2 + \pi rl$$




$$V = \frac{1}{3}Bh$$

$$L.A. = \frac{1}{2}lp$$

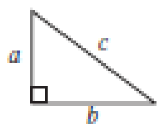
$$S.A. = \frac{1}{2}lp + B$$

### Abbreviations

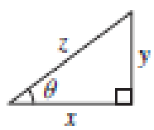
| | |
|---|---|
| Area | *A* |
| Area of Base | *B* |
| Circumference | *C* |
| Lateral Area | *L.A.* |
| Perimeter | *p* |
| Surface Area | *S.A.* |
| Volume | *V* |

# Geometry Formula Sheet
# 2009 *Mathematics Standards of Learning*

## Geometric Formulas



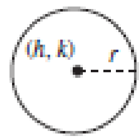

$a^2 + b^2 = c^2$




Sin $\theta = \dfrac{y}{z}$

Cos $\theta = \dfrac{x}{z}$

Tan $\theta = \dfrac{y}{x}$




$(x - h)^2 + (y - k)^2 = r^2$

**Pi**

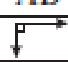
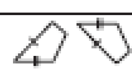
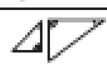
$\pi \approx 3.14$

$\pi \approx \dfrac{22}{7}$

**Quadratic Formula:**

$$x = \frac{^-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0 \text{ and } a \neq 0$$

## Geometric Symbols

| Example | Meaning |
|---|---|
| $m\angle A$ | measure of angle $A$ |
| $AB$ | length of line segment $AB$ |
| $\overrightarrow{AB}$ | ray $AB$ |
|  | right angle |
| $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ | Line $AB$ is parallel to line $CD$. |
| $\overline{AB} \perp \overline{CD}$ | Line segment $AB$ is perpendicular to line segment $CD$. |
| $\angle A \cong \angle B$ | Angle $A$ is congruent to angle $B$. |
| $\triangle ABC \sim \triangle DEF$ | Triangle $ABC$ is similar to triangle $DEF$. |
|  | Similarly marked segments are congruent. |
|  | Similarly marked angles are congruent. |